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ABSTRACT

Various cases of unequal variances and unequal sample sizes from a normal and a skewed population were used to empirically obtain the probability of a Type I error and the power for the permutation t-test as compared to Student's t-test and the Mann-Whitney U-test. Empirical results showed differences for different sample sizes, variance ratios, population sampled, and size of mean of the population. The power of the permutation t-test is very close to or greater than that of Student's t-test for both populations, and the power is large if the large variance accompanied the large mean for the skewed population. (Author)

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The purpose of this research was to investigate empirically the effect of unequal variances on a distribution-free procedure, the permutation t-test, for the two sample case. Specifically, the probability of a Type I error and the power of the permutation t-test against a one-tailed location-shift alternative hypothesis were empirically obtained when one sample was drawn from a population with variance σ_1^2, and the second sample was drawn from a population with variance σ_2^2 $(\sigma_1^2 \neq \sigma_2^2)$, when both populations were one of two types: a normal population and a skewed populatic derived from a χ^2 distribution with three degrees of freedom and skewness measure $\gamma_1 = 1.633$. Comparisons were then made with the empirical probability of a Type I error and the power for Student's t-test and the Mann-Whitney U-test.

The empirical probability of a Type I error and power for the permutation t-test, Student's t-test and the Mann-Whitney U-test were obtained by means of a computer program written to perform the following steps 1) obtain one sample from a population with variance σ_1^2 and a second sample from a population with the same shape as the first but with variance σ_2^2 by means of psuedo-random number generators, 2) add either a null or non-null θ to one of the samples, 3) perform the permutations, rankings and calculations necessary to compute each of the three statistics, 4) record either rejection or acceptance for each of the statistical procedures, and 5) repeat steps 1-4 for 1000 pairs of samples. This procedure resulted in one of the samples being drawn from a population with mean μ and a second sample from a population with mean $\mu + \theta$. When $\theta = 0$, the proportion of rejections was the empirical probability of a

Type I error. When each of several non-null values of a θ (small, medium, or large) was used, the proportion of rejections was the empirical power. The entire procedure was repeated so that results were obtained for all combinations of small and large sample sizes with small and large variances and small and large means (μ or $\mu + \theta$).

The empirical results for the probability of a Type I error (Table 1) for the permutation t-test, Student's t-test and the Mann-Whitney U-test showed no consistent relationships among the three statistical procedures. Comparing the empirical values of the probability of a Type I error for the permutation t-test to those of Student's t-test gave the following frequencies: pt < t, four out of ten, pt > t, four out of ten; and, pt = t, two out of ten. For the Mann-Whitney U-test pt < U, three out of ten, pt > U, four out of ten, and pt = U. The relation of the empirical values of size for the permutation t-test to the theoretical α was very much dependent upon the sample size, ratio of variances and the parent population. For equal sample sizes with a normal population, the following empirical values and theoretical values were observed for various variance ratios sample sizes (3,3), .046 and .05 for 1:3; sample sizes (5,5), .076 and .0476 for 1:3; and, sample sizes (5,5), .054 and .0476 for 1:5. For equal sample sizes with the skewed population, the values of the probability of a Type I error were always very much less than the theoretical values, for example for sample sizes (5,5), variance ratio 1:3, the empirical value was .006 for theoretical value .0476. For both populations when the sample sizes were unequal, the empirical values were always less than the theoretical values if the small variance was with the small sample size (and large variance with large sample size). When the small variance was with the large sample size (and large variance with small sample size), the empirical values

were larger than the theoretical values. Sampling from the skewed population merely accomplished a more drastic reduction or inflation of the empirical values of probability of Type I error.

The empirical results for the power of the permutation t-test (Table 2) showed interesting comparisons which were dependent upon the population sampled. For the normal population with equal sample sizes, the power of the permutation t-test is equal to or greater than that of Student's t-test for nine of eighteen cases and very close for the other nine cases. For unequal sample sizes from the normal population, if the small variance was with the small sample size (and large variance with large sample size) the power values for the permutation t-test were larger than or equal to those for Student's t-test for five of the six cases, but if the large variance was with the small sample size (and small variance with large sample size) none of the six power values for the permutation t-test were larger than or equal to those for Student's t-test. In all cases, the power values of the permutation t-test were close to those for Student's t-test and larger than or equal to those for the Mann-Whitney U-test. Also, there were no consistent differences in power values for the procedures with the large mean ($\mu + \theta$) combined with the small sample or the large sample for the normal population.

For the skewed population, the most outstanding result was the occurrence of very high power values if the large variance accompanied the large mean ($\mu + \theta$) and very low power values if the large variance accompanied the small mean (μ) for all three statistical procedures. This result seemed independent of the sample sizes employed, equal or unequal. Also, the power values for the permutation t-test were greater than or equal to those of the Student's t-test in sixteen of the thirty cases, with the large majority of the cases of larger power for the permutation t-test occurring for the small and

medium values of θ.

Thus, the power of the permutation t-test is generally very close to that of Student's t-test when the population variances are unequal. When the sample sizes also are unequal and proportional to the variances, the power of the permutation t-test is better than that of Student's t-test for both the normal and skewed population. However, if the sample sizes are inversely proportional to the variances, then Student's t-test has higher power than does the permutation t-test, especially for the normal population.

Generally, the present research demonstrated that for the probability of a Type I error, the permutation t-test was reasonably close to that of Student's t-test, even though both were affected by unequal variances and unequal sample sizes, especially for the skewed population.

The choice of one statistical method over another must be based on consideratic of both the probability of a Type I error and the power of the methods. The experimental situation must also be considered in terms of population variances, the population sampled and the sample sizes obtained. The scientific importance of this study is based on the importance of the need of information on the effect of violation of assumptions of non-parametric statistics which are considered substitutes for the carefully researched Student's t-test. Educational researchers may now have comparative information on three statistics for the two-sample case when normality and equal variances assumptions are violated, thus, they may be able to more correctly choose the statistical method which is best for their experimental situation.

Table 1

Probability of a Type I Error for the Permutation t-test (pt), the Mann-Whitney U-test (U) and Student's t-test (t) for Normal and Skewed Populations with Various Ratios of Population Variances

Sample Sizes	Ratio of Variances	Normal Population			Skewed Population			True α
		pt	U	t	pt	U	t	
(3,3)	1:3	.046	.046	.052	.011	.011	.012	.05
(4,5)	1:3	.028	.030	.031	.006	.005	.005	.0317
(4,5)	3:1	.036	.037	.039	.157	.151	.148	.0317
(5,5)	1:3	.076	.070	.075	.006	.007	.006	.0476
(5,5)	1:5	.054	.052	.050	.002	.002	.002	.0476

Table 2

Power of the Permutation t-test (pt), the Mann-Whitney U-test (U) and Student's t-test (t) for Normal and Skewed Populations with Various Ratios of Population Variances for Small, Medium or Large θ

Sample Sizes	Ratio of Variances	θ	Normal Population: Large Mean with Small Sample pt	U	t	Large Mean with Large Sample pt	U	t
(3,3)	1:3	S	.312	.312	.304	.294	.294	.292
		M	.591	.591	.583	.604	.604	.606
		L	.882	.832	.889	.874	.874	.8[illegible]
(4,5)	1:3	S	.237	.273	.278	.254	.262	.257
		M	.562	.551	.563	.555	.535	.545
		L	.875	.852	.875	.877	.854	.871
(4,5)	3:1	S	.333	.319	.344	.321	.307	.336
		M	.612	.586	.639	.615	.562	.636
		L	.891	.862	.903	.901	.883	.921
(5,5)	1:3	S	.339	.320	.342	.289	.272	.291
		M	.625	.585	.623	.534	.549	.563
		L	.894	.861	.893	.890	.864	.890
(5,5)	1:5	S	.321	.224	.312	.307	.279	.305
		M	.609	.555	.612	.607	.545	.602
		L	.901	.845	.902	.883	.833	.885

Table 2 (Continued)

Sample Sizes	Ratio of Variances	θ	Skewed Population: Large Mean with Small Sample: pt	U	t	Large Mean with Large Sample: pt	U	t
(3,3)	1:3	S	.477	.477	.491	.833	.833	.879
		M	.321	.321	.835	.980	.989	.959
		L	.961	.961	.964	.997	.997	.996
(4,5)	1:3	S	.136	.141	.134	.739	.714	.677
		M	.352	.337	.360	.925	.899	.917
		L	.647	.597	.665	.985	.976	.991
(4,5)	3:1	S	.765	.760	.762	.107	.105	.104
		M	.946	.935	.946	.437	.391	.437
		L	.991	.986	.994	.716	.642	.751
(5,5)	1:3	S	.111	.120	.115	.780	.776	.752
		M	.346	.337	.246	.941	.940	.929
		L	.660	.626	.673	.991	.993	.993
(5,5)	1:5	S	.079	.084	.076	.881	.888	.852
		M	.285	.276	.288	.982	.980	.975
		L	.602	.559	.610	.992	.997	.959